Math 202 - Final (Summer 10)

T. Tlas

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- Please answer question 4 on the same sheet of paper on which it is written. Questions 1 and 2 have an extra sheet for you to write your answer on it, while question 3 has two extra sheets for this purpose. Any part of your answer written on the wrong page will not be graded.
- There are 4 problems in total. Most questions have several parts. Make sure that you attempt them all.
- This is a closed book exam and no calculators are allowed.

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Name :

ID # :

Section Number :

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$Q1$	
$Q2$	
$Q3$	
$Q4$	
$TOTAL$	

Problem 1

(15 points each) Solve the following IVPs:

i-

$$y' = ye^x \qquad ; \qquad y(0) = 1$$

ii-

$$y' = \frac{1}{\mathrm{Cos}(x + 2y + 3)} - \frac{1}{2} \qquad ; \qquad y(-3) = 0$$

iii-

$$y'' + y = f(t) \qquad ; \qquad y(0) = 0 \qquad ; \qquad y'(0) = 0$$

where

$$f(t) = \begin{cases} 0 & 0 \leq t < \pi \\ 1 & \pi \leq t < 2\pi \\ 0 & 2\pi \leq t \end{cases}$$

iv-

$$y''' - 3y'' + 2y' = 1 \qquad ; \qquad y(0) = 0 \qquad ; \qquad y'(0) = \frac{1}{2} \qquad ; \qquad y''(0) = 2$$

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ADDITIONAL SHEET FOR PROBLEM 1 ANSWER

Problem 2

(35 points) Find the 10th, 11th, 13th and the 100th coefficients of the series solution (centred at 0) of the following IVP (i.e. find the coefficients of $x^{10}, x^{11}, x^{13}, x^{100}$ of the power series solution of the IVP below)

$$y'' - 2\,x\,y + 8\,y = 0 \qquad ; \qquad y(0) = 1 \qquad y'(0) = 1$$

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ADDITIONAL SHEET FOR PROBLEM 2 ANSWER

Problem 3

(20 points each) Solve the following IVPs:

i-

$$\dot{\mathbf{X}} = \begin{pmatrix} 1 & -1 & 2 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \mathbf{X} \qquad ; \qquad \mathbf{X}(0) = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$$

ii-

$$\begin{aligned} \dot{x} &= y + \delta(t-1) \\ \dot{y} &= -x \end{aligned}$$

$$x(0) = 0 \qquad ; \qquad y(0) = 0$$

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ADDITIONAL SHEET FOR PROBLEM 3 ANSWER

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Problem 4

(15 points) Is there a second-order, linear, ordinary differential equation such that both $y_1(x) = 1$ and $y_2(x) = \mathrm{Sin}(x)$ are solutions of it? In other words, can you find two functions $P(x)$ and $Q(x)$ such that the equation:

$$y'' + P(x)y' + Q(x)y = 0?$$

has $y_1(x)$ and $y_2(x)$ as its solutions?

If yes, give an example. If no, provide a justifcation.

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